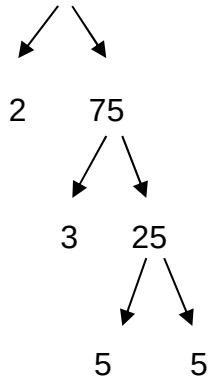
Math 10C Chapter 3 Factors and Products Review Notes

Prime Factorization

- Prime Numbers: Numbers that can only be divided by themselves and 1.
- The first few prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Prime Factorization: Do a factor tree and find all the prime factors of a number.

Ex. 150

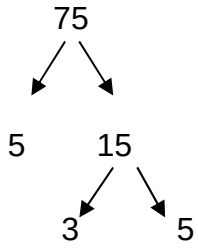


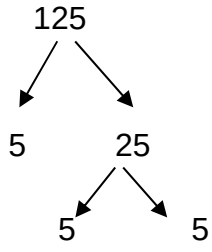
The answer would be: $2 \times 3 \times 5^2$

Greatest Common Factors

To find the greatest common factor, make a factor tree for each number and then find everything that is common and multiply those numbers together to get the GCF.

Ex. Find the GCF of 75 and 125





75: 3 x 5 x 5

125: 5 x 5 x 5

The two numbers have 5 and 5 in common. Therefore, the GCF is
5 X 5 = 25.

Least Common Multiples

To find the least common multiple, do the factor tree. Then, take the highest power of every factor and multiply them together to get the LCM.

So, in our last example:

$75 : 3 \times 5^2$ and $125 : 5^3$

The highest powers of each number are 3 and 5^3. So the LCM is 3 x 5^3 = 375.

Find GCF and LCM of <u>two numbers</u> at once:

The GCF and LCM of 72 and 90

1) Make the following table.

9	72	90
2	8	10
	4	5

2) Divide each number by a common factor.

3) Divide the new numbers by a common factor.
Repeat this process until there is no longer a common factor.

The product of the factors on the left is the GCF:

9 • 2 = 18

The product of the factors on the left AND bottom is the LCM:

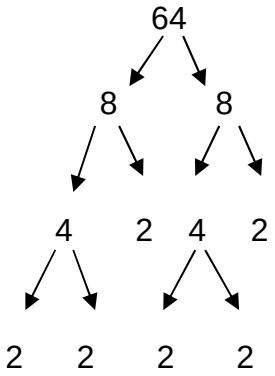
9 • 2 • 4 • 5 = 360

Perfect Squares

- The product of two equal rational numbers.
- Examples: 1 (1x1), 4 (2x2), 9 (3x3), etc.

How can you determine if a number is a perfect square root? Do the factor tree and then see if it **divides into two equal groups**. If it does, it's perfect.

Ex.



Since there are 2 equal identical groups:
2x2x2 and 2x2x2

This is perfect and the square root is 8.

Perfect Cubes

- The product of 3 equal rational numbers
- Examples: 1(1x1x1), 8 (2x2x2), 125 (5x5x5), etc.

How can you determine if a number is a perfect cube root? Do the factor tree and then see if it divides into **three equal groups.** If it does, it's perfect. Look at the last example…..it also divides into 3 equal groups of 2x2 and 2x2 and 2x2. Therefore, the cube root is 4.

Problem Questions:

You may have to do problem questions in which you need to square root or cube root numbers. The formulas you will need are:

Volume of a cube: $V = s^3$	Surface Area of a cube: $6s^2$

Example: Find the edge of the cube with a volume of 5832 cm^3 and then find the surface area.

$V = s^3$

$5832 = s^3$

$\sqrt[3]{5832} = s$

$18cm = s$

$SA = 6s^2$

$SA = 6(18)^2$

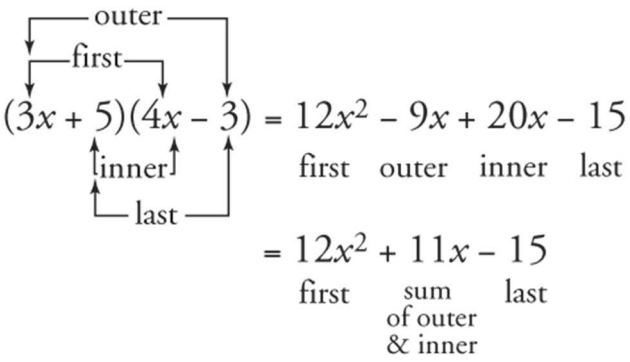
$SA = 1944cm^2$

Simplifying products of polynomials

Simplifying is when we take brackets and we multiply them out.

- The main thing to remember is that when you are multiplying, **coefficients** get multiplied and the **exponents** on the variables get added.
- When you are adding or subtracting, you add/subtract **coefficients** and **leave the variables** the same.
- If there is a number in front of a bracket, distribute it by multiplying it by all the terms in the bracket and then don't write the bracket.
- If there is a negative in front of the bracket, that is coefficient -1 that changes all the signs of the terms in the bracket to their opposite.
- If bracket is squared, write it out and then use FOIL method, or remember pattern for perfect square trinomial.
  $(x+y)^2 = (x+y)(x+y)$

FOIL method for product of two binomials: This involves multiplying **F**irst, **O**utside, **I**nside **L**ast, and then **combining the two middle terms** (add/subtract **coefficients only** of like terms, and leave variable the same)



$$(x+3)(2x-1) = 2x^2 - x + 6x - 3 = 2x^2 + 5x - 3$$

The questions get more complicated by multiplying trinomials by trinomials, putting addition/subtraction signs, etc. Remember that **terms in final answer** always need to go from highest exponent to lowest (descending order by degree), and alphabetically.

$$\begin{aligned}&(2x - 3)(4x^2 + 6x + 9)\\ &= (2x)(4x^2) + (2x)(6x) + (2x)(9) + (-3)(4x^2) + (-3)(6x)\\ &\quad + (-3)(9)\\ &= 8x^3 + 12x^2 + 18x - 12x^2 - 18x - 27\\ &= 8x^3 - 27\end{aligned}$$

$$\begin{aligned}2xy\,(3x-5)^2 &= 2xy\,(9x^2 - 2\cdot 3x\cdot 5 + 5^2)\\ &= 2xy\,(9x^2 - 30x + 25)\\ &= 18x^3y - 60x^2y + 50xy\end{aligned}$$

Factoring - There are many different types of factoring.

1. GCF. Look for the biggest common **number** and **variable**.

   Ex. $5x^2y^3 - 15xy^2 = 5xy^2(xy - 3)$

2. "simple" Trinomials (a=1): these are trinomials that have an invisible 1 in front of the x² terms. To factor these, look for 2 numbers that multiply to the last term and add to the middle term.

   Ex. $a^2 - 7a + 10$ ***we are looking for 2 numbers that multiply to 10 and add to -7. The two numbers are -5 and -2.

   So therefore the answer is: $(a-2)(a-5)$

3. "less simple" Trinomials (a ≠1): these are the trinomials that have a number (coefficient) in front of the x² squared term. Try this method or **guess and check method.** (Or there are lots of others you can find on Internet!)

   ex. $2x^2 - 5x - 7$

   ►A. Break down the first and last terms.

   $2x^2 - 5x - 7$

   2x 7
   1x 1

   B. Put in the signs.

   $2x^2 - 5x - 7$

   2x - 7
   1x + 1

   C. Diagonal multiply to see if the middle term is correct. (2x)(1) = 2x
   (-7)(1x) = -7x

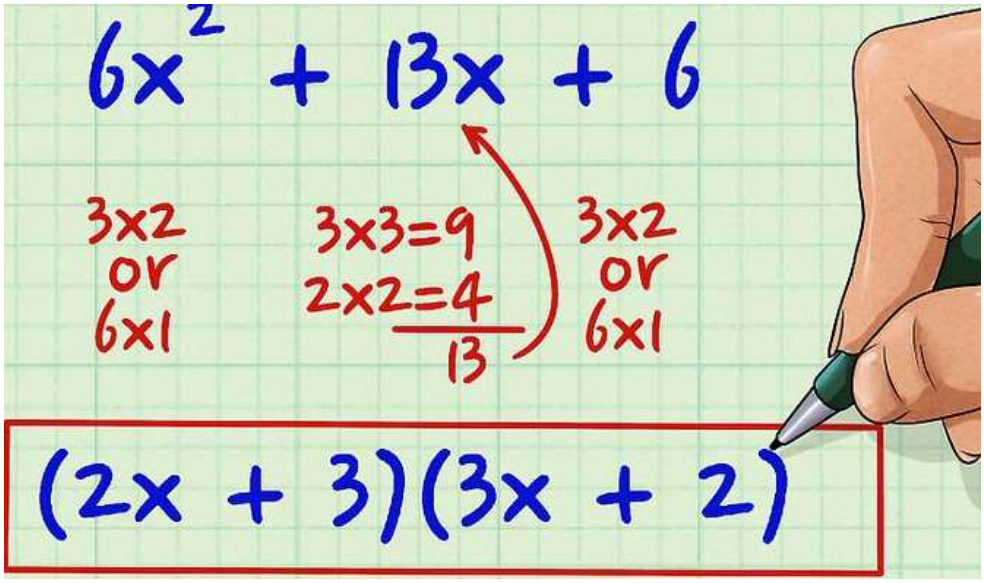
   -7x + 2x = -5x CORRECT!

   D. Draw a horizontal line in between. Above the line goes in the first bracket and below line goes in second. (2x – 7)(x + 1)

If **last term is negative you need two different signs. If last term is **positive**, you need two identical signs.

►Guess and Check Method of Factoring

A. Choose two terms that multiply to make FIRST term and two terms that multiply to make LAST term.



B. Check to see if those numbers work by multiplying only the OUTSIDE and INSIDE of FOIL method. Combine the terms to see if they add/subtract to the middle term.

C. ● If <u>yes,</u> add signs. Multiply all 4 terms (FOIL) and combine middle terms to make sure that these terms and signs work.
● If no, switch position of terms in LAST position of each bracket. Repeat steps A and B. If still no, try two different terms in LAST position. Repeat steps A and B. Again switch LAST two terms if needed. Keep at this until you've tried all factors of LAST term. If still no, then try two different factors of FIRST term. And repeat procedure for trying LAST term. When you find terms that work, go to "YES" above.

4. <u>Perfect Square Trinomials</u>

- These trinomials can be factored into a binomial,squared!
- For something to be a perfect square trinomial, the FIRST and LAST terms must be perfect squares. The <u>middle</u> term must be <u>double the square root</u> of the first and last terms.

Example: $4x^2 - 12x + 9$

Since the first and last terms are perfect squares and the middle term is correct (2x)(3)(2) = 12, we can factor it like so:

$4x^2 - 12x + 9 = (2x - 3)^2$ **the sign will always be the same as the middle term

5. Difference of Squares

   - There can only be two terms and both must be perfect squares
   - There must be a subtraction sign in between.

Example: $x^2 - 25 = (x-5)(x+5)$

These can get more complicated if there are bigger numbers, bigger exponents and GCFs.

How do you know what you should do?

1. If the question says: Prime Factorization, GCF, LCM, Perfect Squares or Perfect Cubes, do the factor tree.
2. If the question says: Simplify, Expand and Multiply, or Expand, use FOIL or the rectangle method.
3. If the question says: Factor, always start by taking out a GCF, if there is one. Then, look at the number of terms:
   - Two terms: check to see if it is a difference of squares
   - Three terms: check to see if it has an invisible one in front of the squared term. If so, it is an easy trinomial and can be factored quickly. If not, then either use Lucky or Decomposition. It could also be a Perfect Square Trinomial, so you could factor that way.